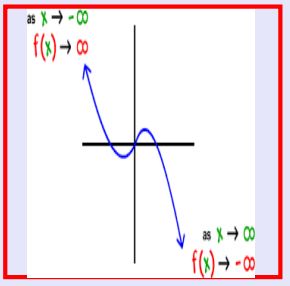
Math 245

Spring 2022

Lecture 33



Divide $\dfrac{4x^2-1}{2x+1}$ using synthetic Division.

L.C. $=1$

$x-K$

Lead. Coef. $=1$

Divide everything by the L.C.

$x+\frac{1}{2}=x-\left(\frac{-1}{2}\right)$

$$\frac{\frac{4}{2}x^2-\frac{1}{2}}{\frac{2}{2}x+\frac{1}{2}}=\frac{2x^2-\frac{1}{2}}{x+\frac{1}{2}}=\frac{2x^2-\frac{1}{2}}{x-\left(\frac{-1}{2}\right)}$$

$K=\frac{-1}{2}$

$$\begin{array}{r|rrr} -\frac{1}{2} & 2 & 0 & -\frac{1}{2} \\ & & -1 & \frac{1}{2} \\ \hline & 2 & -1 & 0 \end{array}$$

0 ← Remainder

$4x^2-1=(2x+1)(2x-1)$

Use Synthetic division to divide

$(6x^3 + x^2 + 10x + 8) \div (3x + 2)$

Must be 1

Divide by 3

$\left(\frac{6}{3}x^3 + \frac{1}{3}x^2 + \frac{10}{3}x + \frac{8}{3}\right) \div \left(\frac{3}{3}x + \frac{2}{3}\right)$

$(2x^3 + \frac{1}{3}x^2 + \frac{10}{3}x + \frac{8}{3}) \div (x + \frac{2}{3})$

$K = -\frac{2}{3}$

$-\frac{2}{3}$	2	$\frac{1}{3}$	$\frac{10}{3}$	$\frac{8}{3}$
		$-\frac{4}{3}$	$\frac{2}{3}$	$-\frac{8}{3}$
	2	-1	4	0 ← Remainder

$\frac{1}{3} + \frac{-4}{3} = \frac{-3}{3} = -1$

$\frac{10}{3} + \frac{2}{3} = \frac{12}{3} = 4$

$\frac{8}{3} + \frac{-8}{3} = 0$

$(2x^2 - x + 4)(3x+2) = 6x^3 + x^2 + 10x + 8$

Use Synthetic division to divide

$(3x^2 - 5x + 4) \div (2x - 1)$

Divide everything by 2.

$\left(\frac{3}{2}x^2 - \frac{5}{2}x + \frac{4}{2}\right) \div \left(\frac{2}{2}x - \frac{1}{2}\right)$ $K = \frac{1}{2}$

$(\frac{3}{2}x^2 - \frac{5}{2}x + 2) \div (x - \frac{1}{2})$

$\frac{1}{2}$	$\frac{3}{2}$	$-\frac{5}{2}$	2
		$\frac{3}{4}$	$\frac{-7}{8}$
	$\frac{3}{2}$	$\frac{-7}{4}$	$\frac{9}{8}$

$-\frac{5}{2} + \frac{3}{4} = \frac{-10}{4} + \frac{3}{4} = \frac{-7}{4}$

$\frac{-7}{8} + 2 = \frac{-7}{8} + \frac{16}{8} = \frac{9}{8}$

Remainder: $\frac{9}{8} \cdot 2 = \frac{9}{4}$

✓ $\frac{3}{2}x - \frac{7}{4} + \frac{\frac{9}{4}}{2x-1}$

verify this by Long division ✓

Use long division to divide $\dfrac{3x^2-5x+4}{2x-1}$

$2x\left[\dfrac{3x}{2}\right]=3x^2$

$-5x-\left(\dfrac{-3x}{2}\right)=$

$-5x+\dfrac{3x}{2}=\dfrac{-10x}{2}+\dfrac{3x}{2}=\dfrac{-7x}{2}$

$2x\left[\dfrac{-7}{4}\right]=\dfrac{-7x}{2}$

$4-\dfrac{7}{4}=\dfrac{16}{4}-\dfrac{7}{4}=\dfrac{9}{4}$

$$2x-1\;\overline{)\;3x^2-5x+4}\qquad \frac{3x}{2}\quad -\frac{7}{4}$$

$$-\left(3x^2-\frac{3x}{2}\right)$$

$$\frac{-7x}{2}+4$$

$$-\left(\frac{-7x}{2}-\frac{7}{4}\right)$$

Remainder $\longrightarrow \dfrac{9}{4}$

$$\frac{3x}{2}-\frac{7}{4}+\frac{\frac{9}{4}}{2x-1}$$

Consider $P(x)=x^3+5x^2-3x-3$

Lead. Coef. $=1$

Const. term $=-3$

List of all possible Zeros are $\dfrac{\pm \text{All Factors of Const.}}{\pm \text{All Factors of L.C.}}$

Solutions / Roots

$\dfrac{\pm \text{All Factors of } 3}{\pm \text{All Factors of } 1}=\dfrac{\pm 1,\pm 3}{\pm 1}\Rightarrow \boxed{\pm 1,\pm 3}$

List of all Possible Zeros

Is 3 a Zero of above $P(x)$?

3	1	5	−3	−3
		3	24	63
	1	8	21	60

Remainder $\neq 0$, 3 is not a Zero.

Is 1 a Zero of above $P(x)$?

1	1	5	−3	−3
		1	6	3
	1	6	3	0

Remainder $=0$, 1 is a Zero of $P(x)$.

Find a list of all possible rational zeros for $P(x) = 2x^3 - 3x^2 - 11x + 6$

$1 \cdot 6 = 6$
$2 \cdot 3 = 6$

Const. Term $= 6 \Rightarrow$ All Factors $\pm 1, \pm 2, \pm 3, \pm 6$

Lead. Coef. $= 2 \Rightarrow$ All Factors $\pm 1, \pm 2$

List of all possible Rational Zeros $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$

$$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$$

Is 3 a zero of above $P(x)$?

3	2	−3	−11	6
		6	9	−6
	2	3	−2	0

Remainder $= 0$, 3 is a zero of $P(x)$

Is $\frac{1}{2}$ a zero of above $P(x)$?

$\frac{1}{2}$	2	−3	−11	6
		1	−1	−6
	2	−2	−12	0

Remainder $= 0$, $\frac{1}{2}$ is a zero of $P(x)$.

$P(x) = 4x^5 - 3x^4 + 6x^3 - 7x^2 + 10$

L.C. $= 4 \Rightarrow$ Factors $\pm 1, \pm 2, \pm 4$

Const. $= 10 \Rightarrow$ Factors $\pm 1, \pm 2, \pm 5, \pm 10$

List of all Possible Rational Zeros $\Rightarrow$ $\dfrac{\text{Factors of Const.}}{\text{Factors of L.C.}}$

$\dfrac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1, \pm 2, \pm 4}$

$\frac{\pm 1}{\pm 1}, \frac{\pm 2}{\pm 1}, \frac{\pm 5}{\pm 1}, \frac{\pm 10}{\pm 1}$

$\frac{\pm 1}{\pm 2}, \frac{\pm 2}{\pm 2}, \frac{\pm 5}{\pm 2}, \frac{\pm 10}{\pm 2}$

$\frac{\pm 1}{\pm 4}, \frac{\pm 2}{\pm 4}, \frac{\pm 5}{\pm 4}, \frac{\pm 10}{\pm 4}$

$\pm 1, \pm 2, \pm 5, \pm 10,$
$\pm \frac{1}{2}, \pm \frac{5}{2},$
$\pm \frac{1}{4}, \pm \frac{5}{4}$

Now we can do synthetic division to find all zeros.

Is 2 a zero?

NO, Remainder $\neq 0$

2	4	−3	6	−7	0	10
		8	10	32	50	100
	4	5	16	25	50	110